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Mass–Diameter and Characteristic-Length Ratio Functions for Orbital Debris

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Over the past several years, the interest in orbital debris has grown because of the hazard it poses to operating satellites. Models of the debris environment depend on accurate assessments of the masses and sizes of debris particles. Utilizing data from several hypervelocity impact tests and other sources, a mass–diameter relationship is determined for debris ranging in size from submillimeter particles to objects as large as derelict rocket bodies and inactive satellites. The accuracy of the mass–diameter relationship is examined and conclusions are reached. The paper also determines that a characteristic diameter of 0.26 cm should be used as a break point between compact and platelike debris models. A function relating the ratios of the debris fragment characteristic lengths also is examined.

Nomenclature

A = least-squares-fit coefficient for mass–diameter distribution
A_c = characteristic area
B = least-squares-fit coefficient for mass–diameter distribution
C = least-squares-fit coefficient for length ratio distribution
D = characteristic diameter for a cylinder
d = diameter, cm
d_1 = lower-bound diameter, cm
d_2 = upper-bound diameter, cm
E = least-squares-fit coefficient for length ratio distribution
F, G, H = coefficients for probability distributions
L = length of cylinder
L_r = characteristic-length ratio
l_1 = longest characteristic length
l_2 = next-longest characteristic length
l_3 = shortest characteristic length
M = computed mass, g
M_1 = mass in smaller-debris equation, g
M_2 = mass in larger-debris equation, g
P = probability of occurrence
X_1 = lower-limit probability function
X_2 = upper-limit probability function
x = $M_{\text{estimated}}/M_{\text{true}}$
Δ = $d_2 - d_1$
κ = probability distribution coefficient
ρ = material density, g/cm^3
σ = standard deviation

Introduction

OVER the past two decades, the interest in orbital debris has grown because of the hazard that it poses to operating satellites. Significant research has been aimed at better understanding the orbital debris environment. Since 1975, considerable effort[1] has been devoted to discerning and understanding the basic relations between mass and size of orbital debris. Many previous studies[2–9] attempting to provide estimates for the masses and sizes of debris particles have concentrated on specific size regimes. Although the method of creation of these debris fragments is related to the mass and size distribution of debris, the authors contend that the mass–diameter and characteristic-length ratio relationships are independent of the fragmentation mechanism. Fragments of a specific size, whether created via a low- or a high-intensity explosion, or a hypervelocity impact, should statistically have similar masses and sizes. Models[10–12] of the debris environment depend on accurate assessments of the masses and sizes of debris particles. This paper combines the data from several studies and presents a mass–diameter function and a characteristic-length ratio function for all sizes of debris particles.

Small Debris Particles

Researchers at the University of Colorado recently attempted to determine a reasonable mass–diameter function for small debris particles.[9] Below a certain size, debris fragments are not limited by the thickness of the originating medium and begin to resemble solid objects instead of twisted plates. Within this size range, researchers have suggested that a spherical model can be used to predict the mass of a debris particle when given its characteristic diameter. Maclay et al.[13] state that "Below a few millimeters, plate thickness is no longer a limiting dimension to a fragment, and a solid, spherical debris becomes a reasonable assumption." Reference 2 suggests that the upper limit for the spherical model should occur at a characteristic diameter no larger than 1 cm, whereas Ref. 12 suggests that this boundary should occur between 0.2 and 0.5 cm, in the case of fragmented delta rocket bodies.

To test the validity of the spherical assumption, a statistical study relating the characteristic diameter to the volume of various debris pieces in which the thickness of the exploded medium was not a limiting factor was required. Because it is difficult to measure submillimeter debris fragments, and it would be expensive to explode a metallic plate several centimeters thick, it was decided that the cheapest, easiest method with which to study debris pieces unlimited by the medium's thickness was to measure the characteristic lengths and volumes of a random sampling of rough-cut rocks. Although it is not contended that unsmoothed rocks and spacecraft break up in an identical fashion, in our experience the shapes taken

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on by unsmoothed rocks closely resemble those of small debris particles. The authors examined a large number of aluminum and steel debris fragments within this size range when measuring the larger fragments within the 5269 (Ref. 5), 5271 (Ref. 6), 5272 (Ref. 7), 6470 (Ref. 8), and Propellant Initiation Program (PIP)[4] tests. Visually, these small fragments appeared similar in shape to the larger rocks that were examined.

Testing Procedure

Twenty rocks that appeared to be small enough to fit inside a graduated cylinder were chosen randomly from a large bed of unsmoothed rocks. One of the rocks was too large to fit in the neck of the graduated cylinder, but the remaining 19 had their characteristic lengths measured, and their volumes determined by measuring the displaced water within the graduated cylinder. The resulting data were tabulated, and a least-squares fitting routine was used to determine the best-fit coefficients. The results indicated that a small particle with a mass equal to 64% of a solid sphere should produce more accurate results than the solid, spherical debris model. By reducing the assorted coefficients related to using a solid sphere as a model to one leading coefficient, the resulting best-fit equation is

$$M = 3.35 \times 10^{-1}(d)^3\rho \tag{1}$$

Figure 1 shows a graph of the data and its best-fit equation.

Error Analysis

The accuracy of the least-squares fit also was examined. The debris particles were normalized in size to a characteristic diameter of one, and their predicted volumes were computed using Eq. (1) and compared with their true volumes. Results reveal that the errors in volume were normally distributed about the expected value for volume. A probability function of the following form describes the distribution in volumes[14]:

$$P = \int_{X_1}^{X_2} \frac{1}{\kappa\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-1}{\kappa}\right)^2\right] dx \tag{2}$$

where κ is 0.21.

Characteristic-Length Ratios

The ratios between the characteristic lengths for the small debris particles also were examined. From a shielding point of view, the relative shape of a debris particle is also important when determining its potential threat. The ratio between the greatest and smallest diameters was computed by assuming that the particle in question was shaped approximately like a cylinder. The characteristic-length ratio L_r was computed as follows:

$$L_r = (L/D) \tag{3}$$

$$D = 2\sqrt{l_2 l_3/\pi} \tag{4}$$

$$L = \frac{\sqrt{\pi} l_1 l_2}{2\sqrt{l_2 l_3}} \tag{5}$$

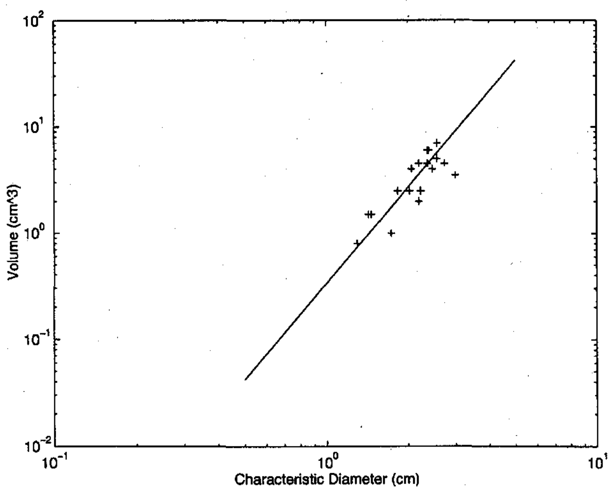

Fig. 1 Small debris particles: volume vs diameter.

The individual lengths are scaled in ascending order. The relative accuracy of this formulation also was examined. The average L_r obtained was 1.92, and the associated σ was 0.786.

Ground-Test Data

The debris fragments from several ground tests have been analyzed at the University of Colorado over the past several years. Hypervelocity impact tests 5269, 5271, 5272, and 6470 were conducted by Arnold Engineering Development Center in Tennessee.[5–8] The fragments resulting from each of these tests were swept up, stored, and later shipped to the University of Colorado for cataloging and analysis. A portion of the fragments resulting from the PIP test were also analyzed at the University of Colorado.[4] A total of 1709 debris fragments was weighed and measured.

General Test Information

The hypervelocity impact tests were initially intended for lethality studies. Metallic cylinders of various constructions were impacted by aluminum projectiles traveling at velocities ranging from 3.38 to 5.5 km/s. The projectiles ranged in mass from 80 to 85 g. The PIP test was a lethality experiment conducted at the U.S. Air Force Phillips Laboratory. It was designed to simulate an explosion event.

Measurement Procedure

Each of the debris fragments in the ground tests analyzed at the University of Colorado were weighed and their characteristic lengths were measured. For these analyses the characteristic diameter d was defined as

$$d = \frac{l_1 + l_2 + l_3}{3} \tag{6}$$

where the three characteristic lengths were the greatest distance across the debris piece in question, the greatest length perpendicular to the first length, and the third length, which forms a normal triad with the other two measurements.

Analysis of Data

Although the initial reports on these data were given in terms of separate mass–diameter functions for each material type,[4–8] this paper reduces these previous efforts to one equation. Data on material types including aluminum, steel, breadboard, glass, foil, ceramic, wire, rubber, and glass were included in this portion of the study. Visual inspection of the metallic debris pieces indicates that there is little difference in their resulting shapes; the differences in the mass–diameter function are due primarily to differences in each material's density. In addition, the nonmetallic pieces appear to have volumes that are comparable to those of metallic pieces with similar diameters. To unify the ground-test data, the masses of the fragments were normalized with respect to their densities. Then, a least-squares fit to the data was performed.[15] The fitting equation used was

$$M/\rho = A(d)^B \tag{7}$$

where A and B were the least-squares coefficients. The resulting equation for determining the mass of a metallic debris particle when given a diameter is

$$M = 1.30 \times 10^{-1}(d)^{2.28}\rho \tag{8}$$

Figure 2 shows the best fit and the data. The diameters of the largest and smallest debris particles measured were 59.67 and 0.26 cm, respectively. The authors suggest that Eq. (8) should be used only within this range.

Error Analysis

An attempt to determine the accuracy of this formulation was undertaken. For each debris piece, the true volume was compared with the predicted volume. The true volume was computed using mass and material density information, and the predicted volume was computed using Eq. (8). The ratio between the predicted volume and the true volume was then computed. Unlike the smaller particles, this error distribution was skewed. The authors determined that the

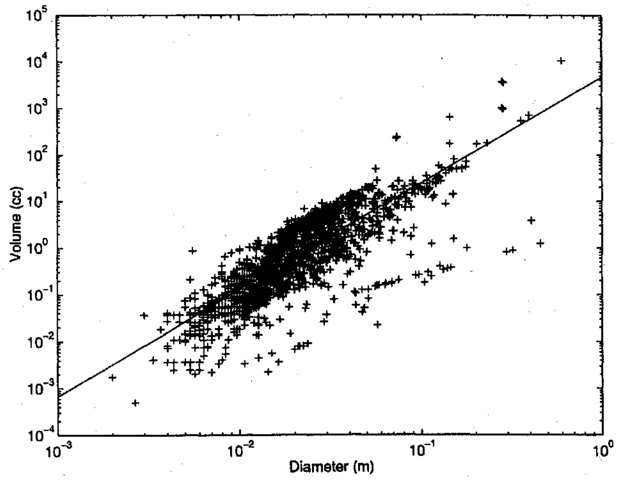

Fig. 2 Ground-test fragments.

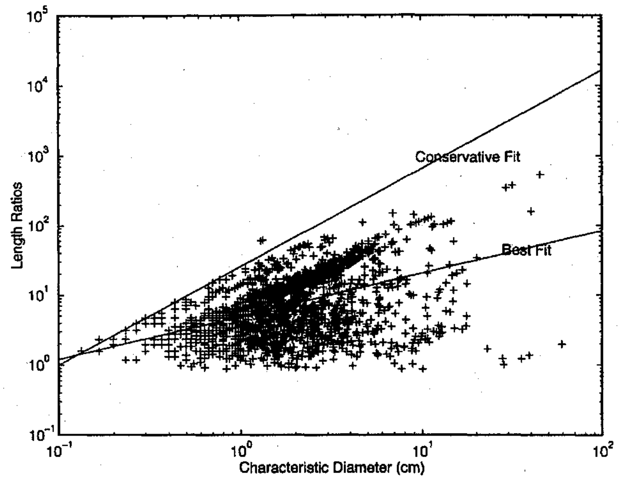

Fig. 3 Ground-test debris: length ratio vs diameter for debris particles and fit.

following probability function describes the discrepancies between the computed and the true volumes:

$$P = \int_{X_1}^{X_2} Fx \exp^{-|Gx-H|} \mathrm{d}x \qquad (9)$$

G and H were chosen such that the shape of the probability distribution given by Eq. (9) matched the shape of the true probability distribution, and F was computed such that the total probability was one. The values for the F, G, and H coefficients were determined to be 1.95, 2.0, and 0.8, respectively.

Characteristic-Length Ratios

The ratios between the characteristic lengths of the ground-test debris particles also were examined. The ground-test data indicate that, unlike the compact debris particles, the characteristic-length ratios are dependent upon the characteristic diameter of the debris object in question. The characteristic-length ratios were computed in the same manner as for the small debris particles. A best fit and a conservative fit were computed to the data set using an equation of the form

$$L_r = C(d)^E \qquad (10)$$

For the best fit, values of 4.94 and 0.613 were determined for C and E, respectively, whereas values of 30 and 1.37 were found for the C and E values of the conservative fit. The two equations are also shown in Fig. 3, with the raw data.

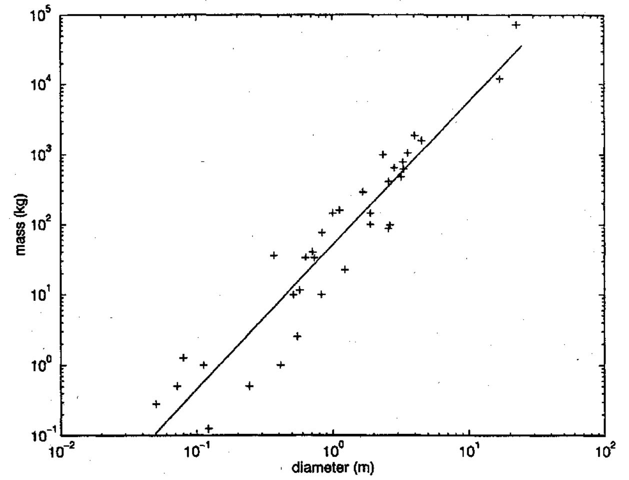

Fig. 4 Kessler and Cour-Palais data[3] and best fit: Kessler's mass-area data.

Orbiting Debris

Kessler and Cour-Palais[3] derived a relationship between mass and size for space debris, using data from orbiting payloads, rocket bodies, and large debris. They used the masses and areas for a few rocket motors and payloads along with the data from the fragments of a Centaur D-1T rocket to arrive at an equation that computed a mass of a debris particle when given an average cross-sectional area. Kessler and Cour-Palais examined 35 objects. For this study, the equation was converted to a mass–diameter equation, where the diameter is assumed to be equal to $[(4A_c)/\pi]^{1/2}$. Figure 4 shows the raw data of Kessler and Cour-Palais,[3] along with the bestfit equation. Because their equation was derived only from debris objects with a characteristic diameter larger than 5 cm, extrapolation into a smaller-size region is not recommended. However, the relation of Kessler and Cour-Palais[3] computes a mass that has no dependency on the material type of the object in question. Reference 12 suggests that the average density of a debris fragment of unknown material type is 4.0 g/cm^3, whereas the average density for a piece of debris originating from a payload is 3.65 g/cm^3. Utilizing information on the mass breakdown of a delta rocket presented in that publication, the authors calculated that the average density for debris originating from a rocket body is 5.8 g/cm^3. Each of the computed masses of Kessler and Cour-Palais was normalized by its predicted density, and least-squares fit to the resulting data was computed. Assuming that these debris objects, payloads, and rocket bodies are relatively normal for orbital debris, the resulting mass–diameter equation is

$$M = 1.18 \times 10^2 (d)^{2.00} \rho \qquad (11)$$

Error Analysis

Again, an attempt to determine the accuracy of this formulation was undertaken. For each debris piece, the true volume was compared with a predicted volume, which was computed using Eq. (11). The ratio between the estimated volume and the true volume was then computed. Unlike the distribution for the smaller particles, this error distribution was skewed. The authors determined that Eq. (9) describes the probability distribution of this best-fit equation with respect to the data. The authors determined that values of 3.62, 2.0, and 0.1 should be used for F, G, and H, respectively.

Boundary Regions

Inspection of the collected data reveals that there are no clear boundaries between each of the data sets. Instead of attempting to integrate all of the disparate data sets into one cohesive data set and solve for one least-squares-fit equation, the authors propose that a set of equations be utilized for determining masses when given size data. For this study, the boundary point between the small debris equation and the ground-test equation was determined by solving for their intersection. The authors determined that this intersection occurred at 0.26 cm, which falls within the range suggested by Ref. 12.

However, no clear boundary point for differentiating between the data of Kessler and Cour-Palais and the ground-test data was determined. The authors propose utilizing a hybrid equation, while within the size regime covered by both equations. Although this formulation is slightly more complex than a single mass–diameter equation, it should produce more-accurate mass predictions. The authors propose that, in this region of overlapping data, an equation of the form described in Eq. (12) be used:

$$M = \left[\frac{(d_2 - d)}{\Delta} \times M_1 + \frac{(d - d_1)}{\Delta} \times M_2\right] \times \rho \qquad (12)$$

A similar formulation also should be valid for the probability density distributions. Table 1 contains the range over which each of these equations should be used.

Debris Densities

To date, very little work examining the average densities of space-debris materials has been completed. Generally, when precise information regarding the material properties of the debris object in question is unavailable, researchers assume that its density is close to that of aluminum. Anderson and Smith[12] also have performed an initial investigation into the average densities of space debris from various sources. Utilizing these results in conjunction with a delta rocket study, the authors suggest that the densities listed in Table 2 should be utilized for computing the masses of various objects that have fragmented while in orbit. In general, these values should be approximately accurate for American spacecraft. However, Russian spacecraft generally are designed using heavier materials, so the densities chosen for debris originating from these sources should be larger.

Table 1 Mass–diameter function

Size range, cm	Equation
$d < 0.26$	(1)
$0.26 < d < 5$	(8)
$5 < d < 59.67$	(8) plus (11) hybrid
$d > 59.67$	(11)

Table 2 Average debris densities for fragments from various sources

Fragment source	Density, g/cm^3
Payloads	3.65
Rocket bodies	5.8
Unknown source	4.0

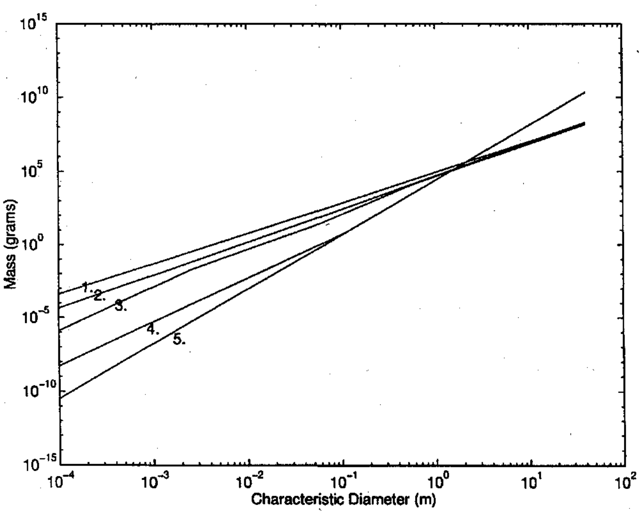

Fig. 5 Comparison of some mass–diameter functions: 1, Ref. 16, satellites; 2, Ref. 2; 3, present study; 4, Ref. 16, debris fragments; and 5, Ref. 17.

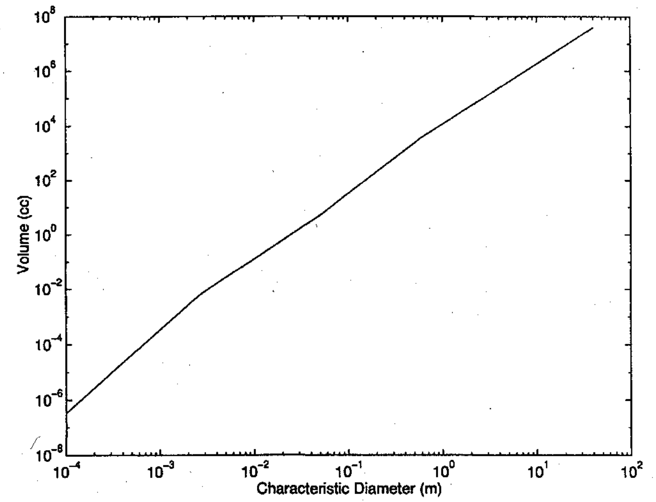

Fig. 6 Complete size regime of metallic debris fragments: volume–diameter function for space debris.

Comparisons with Related Data

Several studies relating debris particle masses to their characteristic sizes have been conducted.[2,16,17] Kessler and co-workers' mass–area function[2] was determined by examining several known objects in orbit. Badhwar and Anz-Meador[16] computed area–mass functions based on orbit decay rates and radar cross-section data. They examined 196 known satellites and approximately 2600 unknown debris fragments resulting from satellite breakups. Jehn's study[17] resulted in a relationship similar to the one presented within this paper. Each of these functions was converted to a mass–diameter format, and was compared with the function proposed within this paper. Figure 5 shows the mass–diameter functions proposed within each of these studies.

Conclusions and Future Work

The authors of this paper have determined a mass–diameter function that is valid for the entire size regime of orbital debris. Unlike previous efforts, this formulation is sensitive to the material type of the debris object in question. The authors also have determined that a characteristic diameter of 0.26 cm should be used as a breakpoint between the compact and the platelike debris objects. The relative accuracy of this equation also was examined.

The authors also have characterized the relative shape of debris particles. Generally, compact debris particles will have smaller characteristic-length ratios, whereas larger debris particles will have larger L/D values. Determining more accurate definitions for the characteristic size of a debris particle is important, because decreasing the error associated with determining the mass of a debris particle will lead to more-accurate space-debris models.

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